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Talking About Math

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Through open and targeted sharing, discussions can build students' understanding of mathematical content.

It only takes five simple words—*How did you get that?*—for math teachers to create opportunities for students to share their thinking. But knowing what to do with students' responses to that question and teaching children to meaningfully participate in discussions can be more daunting. How can teachers keep students from getting lost or disengaged when many different ideas are shared?

Setting goals for mathematics talk and supporting student thinking can help teachers better plan for and facilitate purposeful discussions. Different discussions serve different purposes, and the discussion goal acts as a compass as teachers navigate classroom talk. The goal helps teachers decide what to listen for, which ideas to pursue, and which to highlight (Hiebert, Morris, Berk, & Jansen, 2007). Depending on the ideas at the heart of the lesson, mathematical discussions can be structured as *open strategy sharing* or as *targeted sharing* (Kazemi & Hintz, 2014).

Sometimes a teacher aims to generate many different ideas so students will see a range of possibilities—we call that *open strategy sharing*. In this type of discussion, students contribute different ways to solve the same problem. Such sharing can deepen students' understanding of a repertoire of strategies and show them that different people have different ways of thinking about the same problem.

At other times, a teacher may want to focus the discussion on a particular idea and guide students to*converge* on that idea. We call that *targeted sharing*. This more focused sharing involves specific goals, such as defining and using key terms or concepts correctly, revising an erroneous solution, or making sense of a particular representation. The students listen to and contribute ideas to arrive at consensus. These types of discussions can deepen students' understanding of particular strategies or ideas and help students realize that thinking together can help them understand.

Supporting students as they participate in discussion requires teachers to understand the demands students experience as sharers and listeners (Hintz, 2014). An open strategy share, for example, requires students to make sense of a number of different strategies, size up others' ideas, and determine whether their own solutions are similar to or different from the solutions being discussed. When the discussion zooms in for targeted sharing, students have to listen for and learn to make contributions that add to others' ideas.

Valuing Student Thinking

During classroom discussions, students' ideas are at the center of the talk. However, sharing ideas publicly and in groups can be risky for students. It's not easy for students to express their ideas if there is pressure to be correct and understand everything the first time around. Therefore, it's important that teachers approach discussions with the mind-set that there is logic in students' ideas. Even if a student's solution seems way off base, a teacher can seek out what makes sense about that learner's thinking.

Valuing students' thinking means publicly recognizing their ideas and avoiding singling out only a few students as smart (Cohen, 1994; Featherstone et al., 2011). Teachers want all students to regard themselves as mathematical thinkers and to build their knowledge and skills so they can be successful in math.

The following vignettes from an upper elementary classroom show what we mean by open and targeted sharing. Notice the purposeful moves the teacher makes to accomplish his mathematical goal, support students, and deepen student understanding.

Open Strategy Sharing: Mental Math

Engaging in mental arithmetic is a routine practice in elementary mathematics classrooms. During mental math, a teacher typically poses a computational problem, such as 5 + 2, 12 − 7, 21 × 4, or 96 ÷ 6, and elicits a range of different ways students solved the problem.

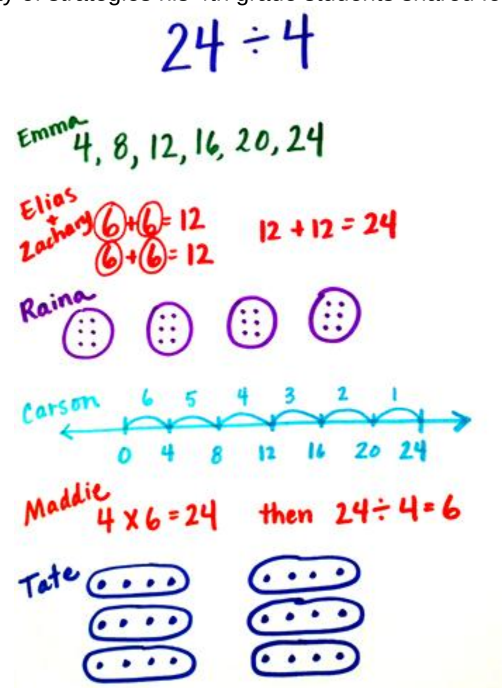
Mr. Foyle wants to understand how his 4th grade students are mentally solving division problems. As he writes the problem 24 ÷ 4 on the easel, he says, "I want you to think about what strategies you have for solving this problem." After a few moments of think time, when it appears every student has at least one solution, he asks students to share their answers. He records every answer on the easel. Students offer three different answers: 6, 4, and 6 remainder 3. Once all the answers have been shared, Mr. Foyle invites students to turn and talk with one another about their solution. As students share with a partner, he circulates around the room, kneeling down to listen. He selects a few solutions for the whole-group discussion (often asking, "May I ask you to share that during our discussion?"). Then he calls the class back together, saying,

We are now going to do a strategy share. I'm going to ask different students to share their strategies. When you share, tell us how you solved the problem and why you solved it that way. As different students explain their thinking, I will write down the strategies. Your job is to listen and make sense of each solution.

Students share several solutions. After each student shares, Mr. Foyle asks, "Did anyone solve it a different way?" Emma describes how she skip counted by 4s six times. Elias and Zachary share that they know 6 plus 6 equals 12, and another 6 plus 6 makes 12 again. They combined the two 12s to see that 24 is made up of four 6s. Raina drew four circles and put dots in each circle one at a time as she counted up to 24 to discover there were six dots in each circle. Carson started at 24 and made six backward jumps of 4 on the open number line. Maddie used her knowledge about 4 × 6 to think about 24 divided by 4. And Tate drew 24 dots and then circled them in groups of 4 to make six groups. Figure 1 shows Mr. Foyle's recording of each solution.

Figure 1. Open Strategy Share

Mr. Foyle recorded a variety of strategies his 4th grade students shared for solving the problem 24 ÷ 4.



During the discussion, the students who originally offered incorrect answers—4 and 6 remainder 3—revise their thinking. Mr. Foyle ends the discussion by recapping the solutions and tells students that in the next few days they will think more about the ideas that emerged.

During this open strategy share, Mr. Foyle made many purposeful moves to accomplish his goal, support students, and deepen student understanding. He began by selecting a problem that would generate many different ideas—which enabled him to work toward his goal of surfacing a wide range of solutions. His questioning strategies ("Did anybody solve it a different way?") also helped elicit different solutions. He valued students' thinking by recording their initial answers and the solutions they shared.

He explicitly supported students in knowing how to share ("Tell us how you solved it and why you solved it that way") and what to listen for ("Your job is to listen and make sense of each solution"). By emphasizing that students should explain not only what they did but also why they did it, he was opening up opportunities for students to deepen their understanding of each solution.

Mr. Foyle can now use what he learned about his students' thinking to plan targeted follow-up discussions. His students might benefit from discussion that generates a justification for why a multiplication fact, 4×6, can be used to solve this division problem (as Maddie did). Such a discussion could help students understand the relationship between multiplication and division and why you can use one operation to think about the other operation. Similarly, further discussion could compare and contrast Raina's solution that makes four groups and Tate's solution that makes groups of four. If lots of students had been making errors, a discussion might reason through a common error and strategies for avoiding it. Mr. Foyle recognizes that he cannot meet all these goals in one discussion, so he decides that students would benefit most from a discussion about Raina's and Tate's solutions.

Targeted Sharing: Compare and Connect

The next day, Mr. Foyle begins the math lesson by saying,

Yesterday, as we were listening to people think about $24 \div 4$, we heard a variety of solutions. Today, I want our discussion to focus on making sense of two particular ways we divided by four and look for similarities in these two solutions. Raina made four groups, and Tate made groups of four. I'm going to put the two strategies in front of us, and I would like you to study them and ask yourself what is similar about them.

After a few moments of think time, he invites students to describe the similarities they notice. Damarias begins, "They're both putting them into groups. But they are opposite of each other."

Mr. Foyle takes up Damarias's thinking, saying, "OK, you are noticing that they are similar because they both put the 24 in groups. Who can add to what Damarias is noticing?"

Javon adds, "They're similar because they have groups, but one has four groups and the other has groups of four."

Turning to the whole class, Mr. Foyle repeats Javon's idea:

One has four groups, and the other has groups of four. I wonder if a problem context could help us here. Imagine these two stories, then turn and talk to your neighbor about what the four means in each of them.

Story A: Our class has 24 students. When we go on our field trip to the zoo, we need to divide into four groups. How many students will be in each group?

Story B: Our class has 24 students. When we go on our field trip to the zoo, we need to divide ourselves into groups with four students in each group. How many groups can we make?

As he listens, Mr. Foyle hears students talking about how the four can be about "how many in a group" or the four could be about "how many groups." He decides to bring the discussion back to Raina's and Tate's strategies and how they are using two different types of division, *partitive* (how many in a group) and*quotative* (how many groups):

Mr. Foyle: I hear people saying that the four in the first story is about how many students are in each group. And that the four in the second story is about how many groups we can make. Let's look at the drawings of Raina's and Tate's

strategies. Which drawing do you think matches which story, and how do you know that? (*After a few moments of wait time*) Gabriel?

Gabriel: I think that Raina's drawing would show a solution for Story A because she made four groups and then put all the students into those four groups. If we look at that drawing, we could see that there would be six students in each group. So, then, I guess, Tate's strategy would show Story B because he made groups of four, and he made six groups.

Mr. Foyle repeats Gabriel's thinking and invites students into another turn and talk to discuss whether they agree or disagree with the matches Gabriel made. This targeted discussion continues as Mr. Foyle maintains a focus on the two solutions and students come to consensus about the multiple meanings of dividing by four.

Before the discussion ends, Mr. Foyle brings back Damarias's language about the strategies being opposite, saying, "Is that what you meant at the beginning of our discussion, Damarias, about the strategies being opposite of each other? Do you want to add to your original thinking now?"

Damarias replies, "Yes, I thought they were opposites because one strategy had four groups and the other strategy had four in each group, but now I'm thinking that they're not really opposite. They're two different ways to divide by four."

Mr. Foyle repeats Damarias's language to culminate the discussion: "That's a clear way to express our thinking, Damarias. There are different ways to divide by four."

This vignette shows many ways Mr. Foyle tries to achieve a mathematical instruction goal while supporting students and deepening their understanding. Rather than eliciting a range of different ways to solve a problem, Mr. Foyle and his students have compared and connected two particular strategies to make meaning of division. Planning for the targeted discussion ahead of time helped Mr. Foyle think about what's mathematically important about how these solutions are similar and different, which informed the way he asked questions and pursued and highlighted certain ideas.

We can also notice the ways Mr. Foyle supported students. For example, as he revoiced Damarias's idea ("You are noticing that they are similar because they both put the 24 in groups. Who can add to what Damarias is noticing?"), he showed that his students' ideas are heard and that each contribution furthers the group's understanding. Similarly, when Mr. Foyle returned to Damarias later in the discussion to bring out the logic in his original statement that the two grouping strategies are "opposite," he was treating Damarias as a sense maker and indicating to students that their ideas can be in progress and that they will have chances to revise or elaborate their thinking over time.

The Goal of Learning

The way teachers and students talk with one another is crucial to what students learn about mathematics and about themselves as doers of mathematics. Teachers play an important role in creating learning opportunities through discussion. As teachers foster productive mathematical discussion, it is important to work toward a mathematical goal while helping students learn how to participate as sharers and listeners.

Different discussions can and should be structured differently, but all discussions, whether open strategy sharing to elicit a wide terrain of ideas or targeted strategy sharing to carefully examine a particular idea, are about achieving a goal and deepening student learning.